



Mathematical Methods Revision Questions - Unit 4 (IA3)

Note: This set of questions closely reflects the simple familiar skills from the Unit 4 Mathematical Methods syllabus as outlined by QCAA. While these questions cover most of the syllabus content, some minor details may have been overlooked and hence not included. This resource is intended to be used as supplementary practice alongside your school's revision material and should not be relied upon as a complete replacement. The primary purpose of this resource is to reinforce core concepts from the QCAA syllabus using mostly simple familiar-style questions.

Unit 4 Topic 1: Further differentiation and applications 3

1. Consider the function:

$$f(x) = x^3 - 6x^2 + 9x + 5$$

- Find the first and second derivatives of the function. **TF**
- Explain what the first derivative tells us about the rate of change of the original function. **TF**
- Identify the intervals where the graph of $f(x)$ is concave up and concave down. **TF**
- Determine any points of inflection **TF**
- Use your graphics calculator to sketch the function, labelling all key features. **TA**

2. A particle moves such that its velocity at time t seconds is given by the function:

$$v(t) = 2t^2 - 4t + 2$$

- Find an expression for the acceleration of the particle. **TF**
- Determine the time at which the acceleration is zero. **TF**
- The particle is at position $s = 3$ metres when $t = 0$. Find the displacement function $s(t)$. **TF**
- Determine the displacement when the acceleration is 0. **TF**

3. Consider the function:

$$f(x) = x^4 - 4x^3$$

- Determine all stationary points **TF**
- Use the second derivative test to classify each stationary as a local maximum, local minimum, or point of inflection. (hint: Remember if $f''(x) < 0$, then maximum, if $f''(x) > 0$, then minimum, and if $f''(x) = 0$, then point of inflection) **TF**
- Sketch the graph of the function, labelling all key features (turning points and inflection points). **TF/TA**
(attempt doing TF and then check answer using graphics calculator)

4. A rectangular paddock is to be fenced on three sides (two widths and one length), with the fourth side bordering a river and requiring no fence. The total length of fencing available is 120 metres.

- Let the width be x . Write an expression for the area in terms of x . **TF**
- Find the value of x that maximises the area. **TF**
- Justify your answer using second derivatives. **TF**
- Calculate the maximum area of the rectangular paddock. **TA**

Unit 4 Topic 2: Trigonometric Functions 2

1. Solve the following problems using trigonometry by first drawing a diagram **TA (ALL)**

- A ladder leans against a wall. The foot of the ladder is 2.5 m from the wall, and the ladder reaches a height of 6 m up the wall. Find the angle the ladder makes with the ground.
- A 12-metre ramp rises vertically to a platform 3.5 m high. Find the angle of elevation of the ramp.
- A surveyor stands 80 m away from the base of a cliff. The angle of elevation to the top of the cliff is 28 degrees. Find the height of the cliff.

2.

a. Use the unit circle to evaluate the following trigonometric expressions. Leave answer as an exact value **TF (ALL)**

(i) $\sin\left(\frac{\pi}{3}\right)$

(ii) $\cos\left(\frac{5\pi}{6}\right)$

(iii) $\tan\left(\frac{\pi}{4}\right)$

(iv) $\sin\left(\frac{7\pi}{6}\right)$

(v) $\cos\left(\frac{3\pi}{2}\right)$

(vi) $\tan\left(\frac{2\pi}{3}\right)$

b. Solve the following equations for $0 \leq \theta \leq 360$ **TF (ALL)**

(i) $\sin \theta = \frac{1}{2}$

(ii) $\cos \theta = -\frac{1}{2}$

(iii) $\tan \theta = 1$

3. In triangle ABC, $a = 7$ cm, $b = 9$ cm, and angle C = 40 degrees.

- Sketch the triangle, labelling all sides and angles. **TF**
- Find the length of side c. **TA**
- Find the size of angle A. **TA**
- Find the size of angle B. **TA**
- Find the area of triangle ABC. **TA**

4. A plane travels 210 km on a bearing of 045°T , then changes direction and travels 100 km on a bearing of 110°T .

- Represent the situation with a labelled diagram. **TF**
- Find the distance between the starting point and the final position. **TA**
- Find the bearing from the final position back to the starting point. **TA**

Unit 4 Topic 3: Discrete random variables 2

1. A light bulb has a 0.88 probability of working correctly.
 - a. Define the Bernoulli distribution for this scenario. **TF**
 - b. What is the probability of a lightbulb not working correctly? **TF**
 - c. Find the mean and variance of the distribution. **TA**

2. A basketball player successfully scores 75% of free throws. He has five attempts. **TA (ALL)**
 - a. Calculate the probability the player scores exactly 4 shots.
 - b. Calculate the probability the player scores at least 4 times.
 - c. Determine the mean, variance, and standard deviation.

3. Solve without a calculator. **TF (ALL)**
 - a. A coin is tossed three times
 - (i) Find $P(3 \text{ heads in a row})$
 - (ii) Find $P(\text{at least one head})$
 - b. A bag has 1 blue and 3 red balls. Two balls are drawn with replacement.
 - (i) Find $P(\text{exactly 1 blue})$
 - (ii) Find $P(\text{at least one blue})$

4. Solve with a calculator **TA (ALL)**
 - a. The probability of winning is 0.6 and 8 games are played.
 - (i) Find $P(\text{at least three wins})$
 - (ii) Find $P(\text{no more than 5 wins})$
 - (iii) Find $P(\text{at least three wins, but no more than five wins})$

4. A machine has a 5% fault rate. In a sample of 20 machines **TA (ALL)**
 - a. Calculate the probability that exactly 2 are faulty.
 - b. Find the probability that at most 2 items are faulty.

Unit 4 Topic 4: Continuous random variables and the normal distribution

1. A probability density function is defined as:

$$f(x) = \begin{cases} k(1 - x^2), & -1 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

- a. Find the value of k. **TF**
- b. Find the expected value $E(X)$ **TA**
- c. Find the variance $\text{Var}(X)$ **TA**
- c. Find $P(X > 0.5)$. **TA**
- d. Find $P(-0.5 < X < 1)$. **TA**

2. Consider the PDF:

$$f(x) = 0.1 \cdot e^{-0.1x}, x \geq 0$$

- a. Find $E(X)$, $\text{Var}(X)$, and $\text{SD}(X)$. **TA**
- b. Find $P(1 < X < 2)$. **TA**

3. Human IQ scores are normally distributed with a mean = 100, SD = 20.

- a. Convert 130 to a z-score. **TF**
- b. Find a minimum IQ score a person would need to have to be in the top 10% of the population **TA**
- c. Find the probability that a person's IQ is between 72 and 115. **TA**
- d. What percentage of IQ scores lie within 1 SD either side of the mean? **TF**
- e. What percentage of IQ scores lie within 2 SDs either side of the mean? **TF**
- f. What percentage of IQ scores lie within 3 SDs either side of the mean? **TF**
- g. What percentage of IQ scores lie outside of being 3SDs either side of the mean? (E.g. What percentage are the lower than -3SD or higher than +3SD) **TF**
- h. Draw a graphical representation for d to g **TF**

4. In a population, the heights of year 10 students are normally distributed with mean 165 cm and SD 6 cm.

- a. What percentage are taller than 171 cm? **TF**
- b. What percentage are between 159 cm and 177 cm? **TF**
- c. What percentage are between 160cm and 180cm? **TA**
- d. What percentage are shorter than 140cm. **TA**
- e. What is the maximum height that would be considered in the lower 25% of year 10 students **TA**
- f. What is the minimum height that would be considered in the top 1% of year 10 students **TA**

Unit 4 Topic 5: Interval Estimates for proportions

1. A survey finds that 72 out of 120 people support a new policy.
 - a. Define the sample proportion (\hat{p}) **TF**
 - b. Given that the population proportion (p) is 0.65, calculate the mean and standard deviation of the sample proportion. **TA**
2. A sample of 500 voters includes 310 who support a candidate.
 - a. Construct a 95% confidence interval for the total population proportion of supporters. **TA**
 - b. Interpret your result in the context of the election (E.g. "We can be 95% confident that..."). **TF**
3. A sample of 150 people in Australia showed that 65 had blonde hair.
 - a. Construct a 90% confidence interval for the total population proportion of Australians with blonde hair. **TA**
 - b. Interpret your result in the context of the survey **TF**
4. A poll shows 55% support with a margin of error $\pm 4\%$.
 - a. Calculate the sample size required to achieve this margin at a 95% confidence level. **TA**
 - b. Explain the relationship between margin of error and width of the confidence interval. **TF**
 - c. Explain the relationship between sample size and the width of the confidence interval. **TF**
5. A study shows that 60% of teenagers enjoy playing video games, with a margin of error of $\pm 2.5\%$
 - a. Calculate the sample size required to achieve this margin of error at a 99% confidence interval **TA**